

Studies of μ -pair and π -pair production at the electron-positron low energy colliders^{*}

S. Jadach

*Institute of Nuclear Physics, Academy of Sciences,
ul. Radzikowskiego 152, 31-342 Cracow, Poland,*

Abstract

Predictions for the radiative return with the muon pair and pion pair final state from KKMC and PHOKHARA Monte Carlo programs are compared and discussed. The case of muon pairs is well understood, especially of the initial state radiation (ISR), where three different second order calculations agree very well. The case of the final state radiation (FSR) requires more tests. Matrix element in KKMC of the EEX type with the incomplete second order NLL corrections is not good enough for the radiative return at $Q^2 < 1\text{GeV}$ with the precision requirement better than 1%. A method of extending the superior CEEX-type matrix element in KKMC to the pion pair final state is described.

Presented at the Epiphany Conference, Cracow, January 2005

IFJPAN-V-05-02

^{*}Supported in part by the EU grant MTKD-CT-2004-510126, in partnership with the CERN Physics Department

1 Introduction

The aim of this contribution is to compare whatever the best we have at hand for evaluation of the initial state radiation (ISR) effect in the process $e^-e^+ \rightarrow \mu^-\mu^+\gamma$ using KKMC [1, 2] and PHOKHARA [3, 4, 5] Monte Carlo programs. The above investigation will be partly extended to the process $e^-e^+ \rightarrow \pi^-\pi^+\gamma$.

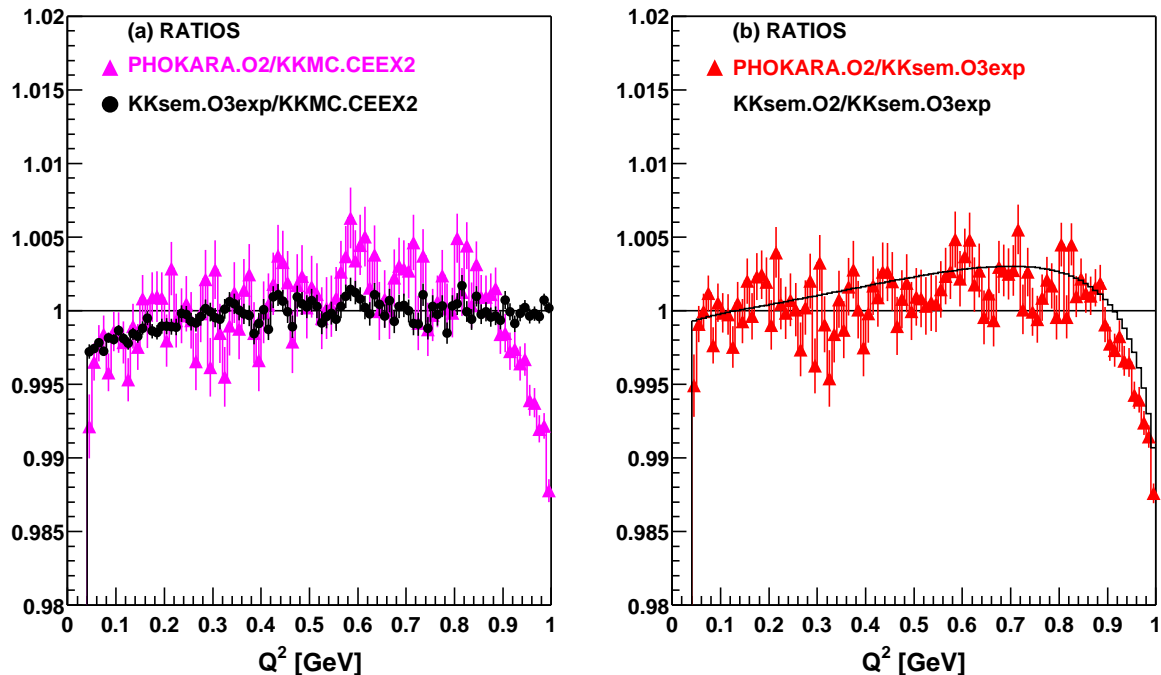


Figure 1: Mu-pair mass (square) spectrum in case of ISR only. $\sqrt{s} = 1.01942\text{GeV}$.

2 ISR in muon pair production

Both KKMC and PHOKHARA programs are full scale MC event generators, which can provide for any experimentally observable distribution. We concentrate however, on the distribution of the squared mass spectrum $Q^2 = s'$ of the muon pair, because this distribution is relevant for the radiative return measurements of $R(s)$, and also because this particular distribution we may compare with the classical semi-analytical calculations. Here we shall also exploit the analytical formulas of ref. [6] (see also [7]), which implement analytical second order ISR calculation of ref. [8] and third order leading-logarithmic (LL) ISR calculation of refs. [9, 10]. The ISR formula of ref. [6] is provided by the KKsem facility of KKMC. In the actual KKsem implementation we use version of the formula where numerically negligible (at least at LEP energies, see ref. [6]) second order NNLL terms are neglected.

It is important to stress from the very beginning that authors of PHOKHARA and KKMC use different terminology to describe Born level matrix element and higher order matrix element. I shall not try to unify terminology or fully explain the differences, referring the reader to original works, like refs. [1, 2] and [3, 4, 5]. Let me explain only very briefly the main differences. The KKMC authors define Born as $e^+e^- \rightarrow f\bar{f}$ without any photon emission and the radiative return is necessarily the first order process with respect to such a Born level. The leading-logarithmic (LL) corrections are of order $\alpha^n L^n$, where $n = 1, 2, 3, \dots, \infty$ is the standard perturbative order, while mass logarithm $L = \ln(s/m_e^2)$ is coming either from the virtual photon correction or the phase space integration over the real photon angle down to zero value. The NLL and NNLL corrections are of order $\alpha^n L^{n-1}$ and $\alpha^n L^{n-2}$ correspondingly. Concerning mass terms, they are routinely neglected in KKMC for the electron (except those which integrate up to a finite correction) while an effort is made to keep all of them for the final state fermions, at least at the Born and the first order level. KKMC implements several variants of the QED matrix elements, which feature different level of higher order and mass term truncation. PHOKHARA authors employ the leading-order (LO) as a name for the process in which one (and only one) photon is emitted in the final state. They name as the next-to-leading-order (NLO) their matrix element with the one-loop corrections and the second real photon. This terminology may seem more adequate to discuss radiative return. However, when trying to match the two terminologies one has to pay attentions to the available phase space of the first real photon. Depending on whether the minimum emission angle is imposed or not, one gets full factor L or not, even at the LO. This affects strongly the relative magnitude of higher order corrections with respect to LO or LL. In this study we generally exclude from the considerations “non-photonic” corrections due to emission of additional lepton pair and vacuum polarization.

In Fig.1 we compare results of KKMC and of PHOKHARA using the best available ISR matrix element in both programs at $\sqrt{s} = 1.01942\text{GeV}$. In KKMC we use second order matrix element with coherent exclusive exponentiation (CEEX) described in refs. [2, 11]. The second order CEEX matrix element has complete next-to-leading-logarithmic (NLL) contributions¹ and complete next-next-to-leading-logarithmic (NNLL) contributions. The magnitude of NLL and NNLL corrections was also examined in a separate studies, see contribution of S. Yost in these proceedings. KKMC includes most of the third order LL contributions by the virtue of exponentiation². On the other hand, PHOKHARA implements complete second order ISR, including complete NLL and NNLL corrections (i.e. singular corrections proportional to $\frac{\alpha}{\pi}m_e^2$ and $\frac{\alpha}{\pi}m_e^4$, which integrate to finite corrections of order $\frac{\alpha}{\pi}$, in the limit $m_e \rightarrow 0$). PHOKHARA does not resum (exponentiate) soft photon contributions to infinite order. It is worth to stress that the two MC calculation, KKMC and PHOKHARA, and semianalytical formula of KKsem of refs. [6, 8] represent set of *three completely independent* second order (using terminology of KKMC) calculation of the ISR in every aspect of calculating QED matrix element and integrating the phase

¹For unpolarized beams, see discussion in ref. [2] related to eq. (128) therein.

²It is also known that exponentiation of the YFS type sums up quite substantial part of third order LL, see refs. [9, 10]

space.

The main comparison of the ISR calculations is shown in Fig.1a, where the distribution $d\sigma/dQ^2$ from KKMC and KKsem agree very well, within 0.2%, except very low Q^2 where they diverge by about 0.3%³, while Fig.1a shows certain addition cross-check. The reason for this discrepancy is not clear. Neglected NNLL in KKsem are a viable candidate, but to confirm this hypothesis one would need more tests. In the same plot we see that PHOKHARA agrees well with KKsem at low Q^2 (aligning with KKsem) and differs by about 0.25% in the central region (we need higher statistics from PHOKHARA to confirm this number) from both KKMC and KKSEM and drops sharply at soft limit, high Q^2 , because of lack of soft photon resummation. In order to understand quantitatively the effect of lack of exponentiation in PHOKHARA we compare its result in Fig.1b with a variant of KKsem in which we switch off exponentiation, i.e. all terms beyond second order are truncated. The smooth curve in Fig.1b representing result of this truncation fits very well PHOKHARA result. In particular, looking into this result, one may think that the deviation of PHOKHARA by 0.25% in the central region is related to its neglect of the third order LL. This conjecture needs more test to be confirmed.

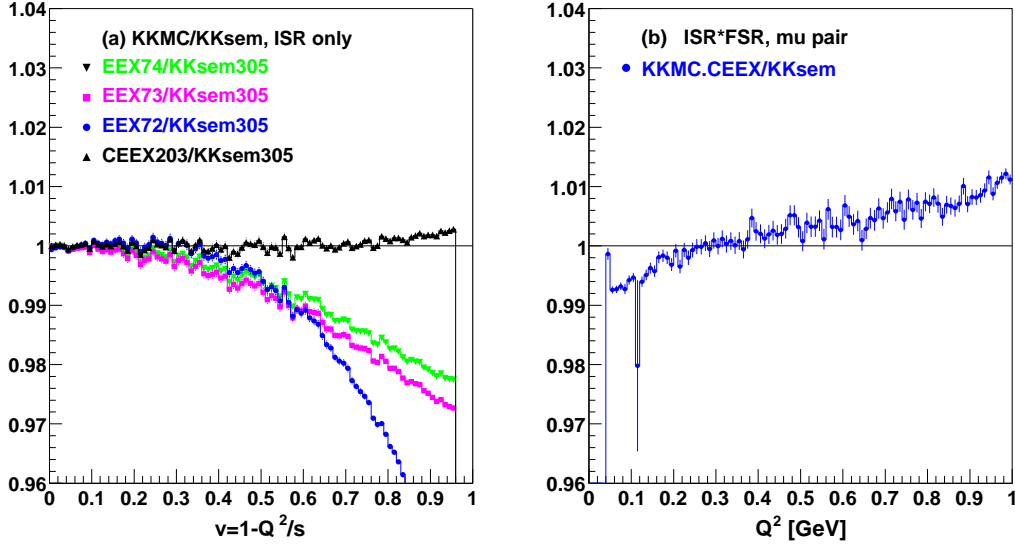


Figure 2: Muon pair mass spectrum from KKMC and KKsem.

We summarize on the results of Fig.1 that KKMC with the second order CEEX matrix element, PHOKHARA with its second order matrix element and KKsem implementing second order analytical calculation agree very well, within the expected range and the pattern of the discrepancies seems to be understood.

In KKMC there is another more primitive QED matrix element denoted as EEX, see ref. [2] for its full description, which follows closely the classical Yennie-Frautschi-

³Note that similar comparison of KKMC and KKsem was done in ref. [2] for LEP energies. At the present lower energy $\sqrt{s} = 1.01942\text{GeV}$ subleading terms are, however, more important.

Suura (YFS) exponentiation scheme and its implementation is limited to first order plus second order LL. In the second order EEX matrix element (contrary to CEEX) the NLL corrections are incomplete. (On the other hand EEX third order LL is complete, while in CEEX it is incomplete.). For technical and historical reasons, see discussion below, EEX type matrix element is used for the production of low energy hadronic final states, for example for pion pair. It is, therefore, important to check how good it is compared to KKMC with more complete coherent exclusive exponentiation (CEEX) matrix element. This is done for the muon final state in Figs. 2. (CEEX is not yet available for π -pairs). In Fig. 2a we see results from KKMC CEEX and several variants of EEX. We are actually plotting $d\sigma/dQ^2$, dividing all results by KKsem of ref. [6], the same as in previous Fig.1. The curves marked EEX72 represent exponentiated EEX matrix element based on complete first order, while EEX73 and EEX74 include also complete second and third LL, while CEEX203⁴ is the same as in Fig. 1. As we see, at low Q^2 , that is for the hard photon emission, results of EEX matrix elements depart from other more complete results by up to 3%! In the ρ region it is different from the KKsem, CEEX KKMC by about 1%. The above result is also consistent with what we have seen in Fig. 1. *EEX is therefore not well suited for the use in the high precision measurements of $R(s)$ using radiative return below $Q^2=1\text{GeV}$.* This result is not very much surprising, as EEX of KKMC has incomplete second order NLL. The observed effect at the low Q^2 is a little bit bigger then what we expected. We have therefore done certain additional tests. We have split EEX results into three components, $\tilde{\beta}_i$, $i = 0, 1, 2$, compared each of them with analytical result of table I of ref. [2], also at $\sqrt{s} = 10\text{GeV}$. We do not show results of these tests here, but the overall pattern of discrepancies seems to be consistent with NLL class of corrections. This additional test indicates also that the main source of the problem is an approximate double real emission matrix element in EEX and not the incomplete virtual corrections. In particular we have included in these tests the complete NLL contribution in $\tilde{\beta}_0$ and $\tilde{\beta}_0$. This did not help! The whole discrepancy seems to result from the use of the LL-approximate matrix element for the double real photon emission in EEX. The above observation is consistent with the older tests in ref. [2] at LEP energies.

2.1 Muon pair, ISR+FSR

Let us not include FSR in the game, again for the muon pair final state. In fact, at low Q^2 the rate of muon pair in radiative return is higher than of π pairs, hence $d\sigma/dQ^2$ of muons can be used as a reference distribution for measuring $R(s)$. It is therefore worth to test FSR in KKMC and to check result for $d\sigma/dQ^2$ once again. In Fig. 2b we show result from KKMC for second order CEEX matrix element in which we include ISR, FSR and its interference. We compare MC result with the semianalytical result of KKsem in which with the same ISR radiative function of ref. [6]. The FSR distribution of ref. [2] features incomplete NLL in KKsem, so it is definitely inferior with respect to ISR counterpart – the complete list of the FSR radiative corrections in KKsem can be found in Table II in ref. [2]. This above semianalytical formula also misses the interference of ISR and FSR,

⁴Indices 203, 74 etc. follow numbering of MC weights in ref. [1].

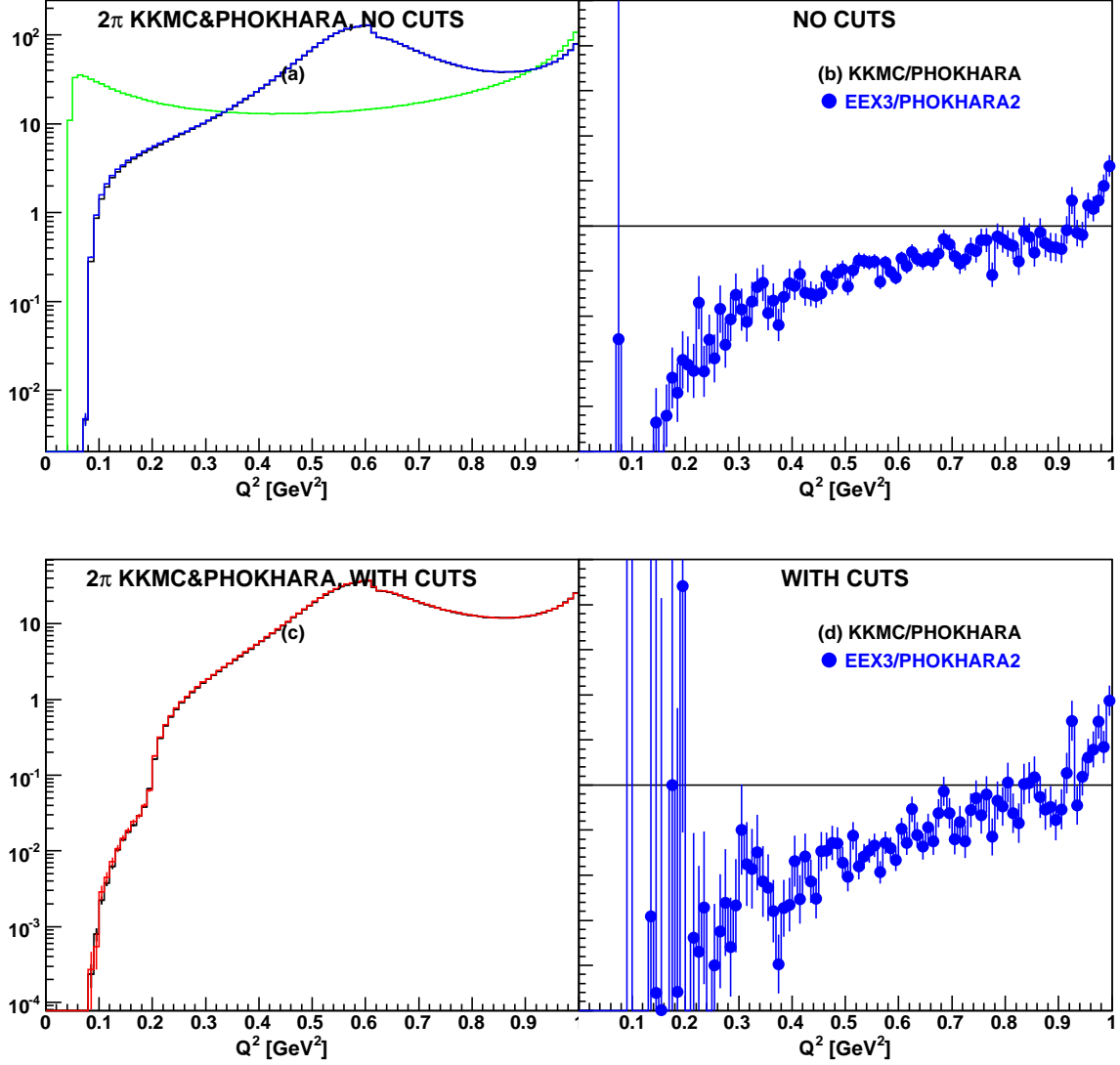


Figure 3: Comparison of PHOKHARA NLO and KKMC with EEX matrix element. ISR only.

which in first order is zero in the inclusive $d\sigma/dQ^2$ so this omission does not harm. In Fig. 2a we see the ratio of the corresponding results from KKMC and KKsem. (NB. PHOKHARA is able to provide result with FSR for muon pairs in the LO, and it would be interesting to include it in the comparison.) This result is rather preliminary and has to be checked. In any case, the agreement better than 1% found all over the dQ^2 range is quite satisfactory as a starting point for further investigation⁵

⁵In one bin we see trace of large weight fluctuation which is probably due to rounding errors. This result was obtained using weighted events. For the MC run with weight-one events this effect would disappear. Such numerical instabilities need further investigation.

3 ISR for $\pi^+\pi^-$ pair production

Let us now switch to low Q^2 π -pair state produced at the radiative return process. In Fig. 3 we compare KKMC with the EEX style matrix element on one hand with PHOKHARA second order (marked as PHOKHARA2) on the other hand. The EEX matrix element is the default one in KKMC, with first order exponentiation, the completed second and third order LL (EEX74). We limit ourselves to ISR only. The results of Figs. 3a-b are obtained without any cutoffs. In Fig. 3a we show the actual distributions, including also the distribution for the muon pairs. We see that for $Q^2 < 0.33$ the muon-pair cross section is bigger than that of π -pair. The ratio PHOKHARA/KKMC is not so well understood as the analogous results for the muon pair shown in the previous section. The discrepancy at high Q^2 we attribute to lack of exponentiation in PHOKHARA⁶ while another larger discrepancy at low Q^2 is most likely due to incompleteness of second order NLL in EEX matrix element on KKMC, and it corresponds to deviation which was already seen in Fig. 2a. In Fig. 3c-d we show the analogous results for relatively mild cut on photon momentum, where photon is defined as a “missing four momentum” calculated knowing pion momenta and beam momenta. We ask for the momentum of such a “collective unseen photon” to be directed below 15° from the beam and to have at least 10MeV of energy. For each π we require that it is situated in wide angles, eg. separated by more than 40° from each beam. Results in Fig. 3c-d look quite similar, except that the discrepancy between PHOKHARA and EEX KKMC is bigger (we need better statistics from PHOKHARA to see it more clearly). This can be attributed to the fact that the leading logarithm L due to real emission is diminished by the cut on the photon angle with respect to beams.

3.1 How to extend CEEX ISR to hadronic final states?

In the following we show that the superior CEEX ISR matrix element can be extended to hadronic final states at low Q^2 , like pion pair. This can be done provided we have some decent modeling of the hadronic final state in terms of the corresponding formfactor. In view of its practical importance, let us elaborate on this point.

⁶This does not hinder practical applications of PHOKHARA for radiative return measurements, which concentrate at lower Q^2 .

In CEEX Born amplitude for $ee \rightarrow \mu\mu$ is defined as a four-spinor tensor

$$\begin{aligned}
\mathfrak{B} \left(\begin{smallmatrix} p \\ \lambda \end{smallmatrix}; X \right) &= \mathfrak{B} \left(\begin{smallmatrix} p_a p_b p_c p_d \\ \lambda_a \lambda_b \lambda_c \lambda_d \end{smallmatrix}; X \right) = \mathfrak{B} \left[\begin{smallmatrix} p_b p_a \\ \lambda_b \lambda_a \end{smallmatrix} \right] \left[\begin{smallmatrix} p_c p_d \\ \lambda_c \lambda_d \end{smallmatrix} \right] (X) = \mathfrak{B}_{[ba][cd]}(X) = \\
&= ie^2 \sum_{B=\gamma, Z} \Pi_B^{\mu\nu}(X) (G_{e,\mu}^B)_{[ba]} (G_{f,\nu}^B)_{[cd]} H_B = \sum_{B=\gamma, Z} \mathfrak{B}_{[bc][cd]}^B(X), \\
(G_{e,\mu}^B)_{[ba]} &\equiv \bar{v}(p_b, \lambda_b) G_{e,\mu}^B u(p_a, \lambda_a), \quad (G_{f,\mu}^B)_{[cd]} \equiv \bar{u}(p_c, \lambda_c) G_{f,\mu}^B v(p_d, \lambda_d), \\
G_{e,\mu}^B &= \gamma_\mu \sum_{\lambda=\pm} \omega_\lambda g_\lambda^{B,e}, \quad G_{f,\mu}^B = \gamma_\mu \sum_{\lambda=\pm} \omega_\lambda g_\lambda^{B,f}, \quad \omega_\lambda = \frac{1}{2}(1 + \lambda\gamma_5), \\
\Pi_B^{\mu\nu}(X) &= \frac{g^{\mu\nu}}{X^2 - M_B^2 + i\Gamma_B X^2/M_B},
\end{aligned} \tag{1}$$

and it enters as a basic building block in every spin amplitude in the CEEX scheme, with arbitrary number of photons. See eq. (43) in ref. [2] for notation. The above Born is calculated using Chisholm identity and replaced with the bi-spinor objects of the Kleiss-Stirling method.

In case of hadronic final state the structure of the Born amplitude is

$$\mathfrak{B}_{[ba]}^\mu(X) J_\mu(X, q_i), \quad J_\mu(X, q_i) X^\mu = 0, \tag{2}$$

where q_i are momenta of the final state hadrons, $X = \sum q_i$, and

$$\mathfrak{B}_{[ba]}^\nu(X) = ie^2 \sum_{B=\gamma, Z} H_B (G_{e,\mu}^B)_{[ba]} \Pi_B^{\mu\nu}(X) = \sum_{B=\gamma, Z} \mathfrak{B}_{[bc]}^{B\mu}(X). \tag{3}$$

In the rest frame of X one has $J^\mu = (0, \vec{J})$ and we may split J into difference of the two massless four-vectors $J^\mu = J_+^\mu - J_-^\mu$. In the arbitrary reference frame the above prescription extends as follows⁷

$$J_\pm^\mu = \frac{1}{2\sqrt{X^2}} (\sqrt{-J^2} X^\mu \pm \sqrt{X^2} J^\mu). \tag{4}$$

Each of the two corresponding components in \mathfrak{B} can be expressed in terms of the of the Kleiss-Stirling bi-spinors $s_\pm(p_1, p_2)$, see eqs. (A4-A6) in ref. [2]. This can be done using completeness relation for $\bar{v}(p_b, \lambda_b) \gamma_\nu J_\pm^\nu u(p_a, \lambda_a)$ taking advantage of $J_\pm^2 = 0$.

Note that the above CEEX implementation requires that we parametrize the production amplitude of the each final hadronic state one by one in terms of the formfactors in a completely exclusive manner. However, this is necessary anyway for good phenomenological description of these often resonant low energy hadronic states. We conclude that CEEX can be used for ISR for low energy hadron production. The question is only how much programming it will be and who will do it.

⁷The author would like to thank J. Kuehn for suggesting him this solution.

3.2 Conclusions

The case of radiative return with the muon pair final state is well understood, especially for the ISR where three different second order calculations agree very well. The case with FSR requires more tests. Matrix element EEX of KKMC with the incomplete second order NLL is not good in the radiative return at $Q^2 < 1\text{GeV}$ for precision requirement better than 1%. Method of porting CEEX matrix element of KKMC to pion pair final state is outlined.

Acknowledgments

This work is partly supported by TARI Contract No. RII3-CT-2004-506078. The author would like to thank J. Kuehn and A. Denig for useful discussion. Warm hospitality at TTP Karlsruhe University and INFN Frascati, where part of this work was done is also acknowledged.

References

- [1] S. Jadach, B. F. L. Ward, and Z. Was, *Comput. Phys. Commun.* **130** (2000) 260–325, [hep-ph/9912214](#).
- [2] S. Jadach, B. F. L. Ward, and Z. Was, *Phys. Rev.* **D63** (2001) 113009, [hep-ph/0006359](#).
- [3] G. Rodrigo, H. Czyz, J. H. Kuhn, and M. Szopa, *Eur. Phys. J.* **C24** (2002) 71–82, [hep-ph/0112184](#).
- [4] H. Czyz, A. Grzelinska, J. H. Kuhn, and G. Rodrigo, *Eur. Phys. J.* **C27** (2003) 563–575, [hep-ph/0212225](#).
- [5] J. H. Kuhn and G. Rodrigo, *Eur. Phys. J.* **C25** (2002) 215–222, [hep-ph/0204283](#).
- [6] S. Jadach, M. Skrzypek, and B. F. L. Ward, *Phys. Lett.* **B257** (1991) 173–178.
- [7] S. Jadach, M. Skrzypek, and M. Martinez, *Phys. Lett.* **B280** (1992) 129–136.
- [8] F. Berends, G. Burgers, and W. V. Neerven, *Phys. Lett.* **177** (1986) 1191.
- [9] M. Skrzypek, *Acta Phys. Polon.* **B23** (1992) 135–172.
- [10] M. Skrzypek and S. Jadach, *Z. Phys.* **C49** (1991) 577–584.
- [11] S. Jadach, B. F. L. Ward, and Z. Was, *Phys. Lett.* **B449** (1999) 97–108, [hep-ph/9905453](#).